# Math 202
# Assignment 1: Hyperbolic Fuctions

**The due date for this assignment is .....**
Reading assignment: Section 3.11


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1. Find the numerical value of each expression

   **(a)** $\sinh(\ln 2)$

   **(b)** $\sinh 2$

2. Prove the identity

   - $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$
   - $\coth^2 x - 1 = \operatorname{csc} h^2 x$
   - $\tanh(\ln x) = \dfrac{x^2-1}{x^2+1}$

3. If $coshx = \dfrac{5}{3}$ and $x \geq 0$, find the values of the other hyperbolic functions at $x$.

4. Find the derivatives. Simplify where possible.

   - $f(t) = \operatorname{sec} h^2(e^t)$
   - $y = \sinh(\cosh x)$
   - $G(x) = \dfrac{1-\cosh x}{1+\cosh x}$
   - $y = x \tanh^{-1} x + \ln\sqrt{1-x^2}$
   - $y = x \sinh^{-1}\left(\dfrac{x}{3}\right) - \sqrt{9+x^2}$

# Math 202
# Assignment 2: Indeterminate Forms and L'Hospital's Rule

**The due date for this assignment is .....**
Reading assignment: Section 4.4


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1. Find the limit. Use L'hospital's Rule where appropriate. If L'hospital's Rule doesn't apply, explain why

   **(a)** $\lim\limits_{x\to 1} \dfrac{x^a - 1}{x^b - 1}$

   **(b)** $\lim\limits_{x\to 0} \dfrac{\sqrt{1+2x} - \sqrt{1-4x}}{x}$

   **(c)** $\lim\limits_{x\to 0} \dfrac{x\ 3^x}{3^x - 1}$

   **(d)** $\lim\limits_{x\to 1} \dfrac{x^a - ax + a - 1}{(x-1)^2}$

   **(e)** $\lim\limits_{x\to 0} \cot(2x)\,\sin(6x)$

   **(f)** $\lim\limits_{x\to \infty} x^3 e^{-x^2}$

   **(g)** $\lim\limits_{x\to 1^+} \ln(x)\,\tan\left(\dfrac{\pi x}{2}\right)$

   **(h)** $\lim\limits_{x\to 1} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right)$

   **(i)** $\lim\limits_{x\to \infty} (x - \ln x)$

   **(j)** $\lim\limits_{x\to 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)]$

   **(k)** $\lim\limits_{x\to 0^+} (4x+1)^{\cot x}$

   **(l)** $\lim\limits_{x\to 0^+} (\cos x)^{1/x^2}$

# Math 202

# Assignment 3: Antiderivatives

**The due date for this assignment is .....**
Reading assignment: Section 4.9


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1. Find the most general antiderivatives of the function (check your answer by differentiation)

   **(a)** $f(x) = (x+1)(2x-1)$

   **(b)** $f(x) = \sqrt{2}$

   **(c)** $f(x) = \dfrac{1+t+t^2}{\sqrt{t}}$

2. Find $f$

   - $f'''(t) = \cos t$
   - $f'(t) = \dfrac{4}{1+t^2}, \quad f(1) = 0$
   - $f'(t) = 2\cos t + \sec^2 t, \quad -\dfrac{\pi}{2} < t < \dfrac{\pi}{2}, \quad f\left(\dfrac{\pi}{3}\right) = 4$
   - $f'(x) = x^{-1/3}, \quad f(1) = 1, \quad f(-1) = -1$
   - $f''(x) = x^{-2}, \quad x > 0, \quad f(1) = 0, \quad f(2) = 0$

3. A particle is moving with the given data. Find the position of the particle

   - $v(t) = \sin t - \cos t, \quad s(0) = 0$
   - $a(t) = 2t + 1, \quad s(0) = 3, v(0) = -2$

# Math 202

# Assignment 4: The Definite Integral

**The due date for this assignment is .....**
Reading assignment: Section 5.2


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1. Evaluate the integral by interpreting it in terms of areas

   **(a)** $\int_{-1}^{2} (1-x)\, dx$

   **(b)** $\int_{-1}^{2} |x|\, dx$

2. Write as a single integral in the form $\int_a^b f(x)\, dx$ :

$$\int_{-2}^{2} f(x)\, dx + \int_{2}^{5} f(x)\, dx - \int_{-2}^{-1} f(x)\, dx$$

3. If $\int_1^5 f(x)\, dx = 12$ and $\int_4^5 f(x)\, dx = 3.6$, find $\int_1^4 f(x)\, dx$

4. Find $\int_0^5 f(x)\, dx$ if

$$f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$$

5. Use property 8 to estimate the value of the integral

   - $\int_1^4 \sqrt{x}\, dx$
   - $\int_{\pi/4}^{\pi/3} \tan x\, dx$

# Math 202

# Assignment 5: The Fundamental Theorem of Calculus

**The due date for this assignment is .....**

Reading assignment: Section 5.3


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1. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function

   **(a)** $h(x) = \int_1^{e^x} \ln(t)\, dt$

   **(b)** $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2}\, du$

2. Evaluate the integral.

   **(a)** $\int_1^2 (1+2y)^2\, dy$

   **(b)** $\int_0^1 (x^e + e^x)\, dx$

   **(c)** $\int_{-1}^1 e^{u+1}\, du$

   **(d)** $\int_0^{\pi} f(x)\, dx$ where

$$f(x) = \begin{cases} \sin x & \text{if } 0 \le x < \pi\backslash 2 \\ \cos x & \text{if } \pi\backslash 2 \le x \le \pi \end{cases}$$

3. What is wrong with the equation?

   - $\int_{-2}^1 x^{-4} dx = \frac{x^{-3}}{-3}]_{-2}^1 = -\frac{3}{8}$

4. Find the derivative of the function

   - $y = \int_{\cos x}^{\sin x} \ln(1+2v)\, dv$

# Math 202

# Assignment 6: Indefinite Integrals and The Net Change

**The due date for this assignment is .....**
Reading assignment: Section 5.4


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1. Find the general indefinite integral.

   **(a)** $\int (1 + \tan^2 \alpha)\, d\alpha$

2. Evaluate the integral.

   **(a)** $\int_0^1 (x^{10} + 10^x)\, dx$

   **(b)** $\int_0^{\pi\backslash 4} \dfrac{1 + \cos^2 \theta}{\cos^2 \theta}\, d\theta$

   **(c)** $\int_0^{\sqrt{3}\backslash 2} \dfrac{dr}{\sqrt{1 - r^2}}$

   **(d)** $\int_0^{1\backslash\sqrt{3}} \dfrac{t^2 - 1}{t^4 - 1}\, dt$

   **(e)** $\int_{-1}^{2} |\, x - 2 |\, x \,|\, |\, dx$

# Math 202

# Assignment 7: The Substitution Rule

**The due date for this assignment is .....**
Reading assignment: Section 5.5


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1. Evaluate the indefinite integral.

   **(a)** $\int 5^t \sin(5^t)\, dt$

   **(b)** $\int \dfrac{\cos\theta}{\sin^2\theta}\, d\theta$

   **(c)** $\int \dfrac{\sin 2x}{1+\cos^2 x}\, dx$

   **(d)** $\int \dfrac{\sin x}{1+\cos^2 x}\, dx$

   **(e)** $\int \dfrac{dx}{\sqrt{1-x^2}\sin^{-1} x}$

   **(f)** $\int \dfrac{x}{1+x^4}\, dx$

   **(g)** $\int \dfrac{1+x}{1+x^2}\, dx$

2. Evaluate the definite integral.

   **(a)** $\int_{-\pi\backslash 4}^{\pi\backslash 4} (x^3 + x^4\tan x)\, dx$

   **(b)** $\int_1^2 x\sqrt{x-1}\, dx$

   **(c)** $\int_0^1 \dfrac{dx}{(1+\sqrt{x})^4}$

# Math 202

# Assignment 8: Integration by Parts

**The due date for this assignment is .....**
Reading assignment: Section 7.1


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1. Evaluate the integral.

   **(a)** $\displaystyle\int (x^2 + 2x)\cos x \; dx$

   **(b)** $\displaystyle\int \ln\left(\sqrt[3]{x}\right) \; dx$

   **(c)** $\displaystyle\int x \; 2^x \; dx$

   **(d)** $\displaystyle\int \frac{x \; e^{2x}}{(1+2x)^2} \; dx$

   **(e)** $\displaystyle\int_0^1 \frac{y}{e^{2y}} \; dy$

   **(f)** $\displaystyle\int \cos x \, \ln(\sin x) \; dx$

   **(g)** $\displaystyle\int_1^2 x^4 (\ln x)^2 \; dx$

2. First make a substitution and then use intgration by parts to evaluate the integral.

   **(a)** $\displaystyle\int \cos\sqrt{x} \; dx$

   **(b)** $\displaystyle\int_{\sqrt{\pi\backslash 2}}^{\sqrt{\pi}} \theta^3 \cos\left(\theta^2\right) \; d\theta$

   **(c)** $\displaystyle\int x \ln(1+x) \; dx$

# Math 202

# Assignment 9: Trigonometric Integrals

**The due date for this assignment is .....**
Reading assignment: Section 7.2


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1. Evaluate the integral.

   **(a)** $\int_0^{\pi} \cos^4(2x)\, dx$

   **(b)** $\int_0^{\pi\backslash 2} \sin^2 x \cos^2 x\, dx$

   **(c)** $\int t \sin^2 t\, dt$

   **(d)** $\int \dfrac{\cos^5 x}{\sqrt{\sin x}}\, dx$

   **(e)** $\int \cos^2 x \tan^3 x\, dx$

   **(f)** $\int \tan^5 x\, dx$

   **(g)** $\int x \sec x \tan x\, dx$

   **(h)** $\int_{\pi\backslash 4}^{\pi\backslash 2} \cot^5 \phi \csc^3 \phi\, d\phi$

   **(i)** $\int \csc x\, dx$

   **(j)** $\int_0^{\pi\backslash 6} \sqrt{1+\cos 2x}\, dx$

   **(k)** $\int x \tan^2 x\, dx$

# Math 202

# Assignment 10: Trigonometric Substitution

**The due date for this assignment is .....**
Reading assignment: Section 7.3


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1. Evaluate the integral.

   **(a)** $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

   **(b)** $\displaystyle\int \frac{\sqrt{1+x^2}}{x}\,dx$

   **(c)** $\displaystyle\int_0^{0.6} \frac{x^2}{\sqrt{9-25x^2}}dx$

   **(d)** $\displaystyle\int \frac{x}{\sqrt{x^2+x+1}}\,dx$

   **(e)** $\displaystyle\int \sqrt{x^2+2x}\,dx$

   **(f)** $\displaystyle\int x\sqrt{1-x^4}\,dx$

# Math 202

# Assignment 11: Integration of Rational Function by Partial Fractions

**The due date for this assignment is .....**

Reading assignment: Section 7.4


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1. Evaluate the integral.

   **(a)** $\displaystyle\int \frac{4x}{x^3+x^2+x+1}\,dx$

   **(b)** $\displaystyle\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)}\,dx$

   **(c)** $\displaystyle\int \frac{x+4}{x^2+2x+5}dx$

   **(d)** $\displaystyle\int \frac{1}{x^3-1}\,dx$

2. Make a substitution to express the integrand as a rational function and then evaluate the integral.

   **(a)** $\displaystyle\int \frac{\sqrt{x+1}}{x}\,dx$

   **(b)** $\displaystyle\int \frac{dx}{x^2+x\sqrt{x}}$

   **(c)** $\displaystyle\int \frac{x^3}{\sqrt[3]{x^2+1}}\,dx$

   **(d)** $\displaystyle\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}\,dx$ [Hint: Substitute $u=\sqrt[6]{x}$]

   **(e)** $\displaystyle\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$

   **(f)** $\displaystyle\int \frac{\sec^2 x}{\tan^2 x+3\tan x+2}\,dx$

   **(g)** $\displaystyle\int \frac{dx}{1+e^x}$

# Math 202

# Assignment 12: Strategy for Integration

**The due date for this assignment is .....**

Reading assignment: Section 7.5


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1. Evaluate the integral.

   **(a)** $\displaystyle\int e^{x+e^x}\,dx$

   **(b)** $\displaystyle\int \frac{1}{1+e^x}\,dx$

   **(c)** $\displaystyle\int \frac{\sec x\,\tan x}{\sec^2 x-\sec x}dx$

   **(d)** $\displaystyle\int \theta\,\tan^2\theta\,d\theta$

   **(e)** $\displaystyle\int \frac{\sqrt{x}}{1+x^3}\,dx$

   **(f)** $\displaystyle\int x^5\,e^{-x^3}\,dx$

   **(g)** $\displaystyle\int x^3\,(x-1)^{-4}\,dx$

   **(h)** $\displaystyle\int \cos x\,\cos^3(\sin x)\,dx$

   **(i)** $\displaystyle\int \frac{\sin 2x}{1+\cos^4 x}\,dx$

   **(j)** $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$

   **(k)** $\displaystyle\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2}\,dx$

   **(l)** $\displaystyle\int \frac{xe^x}{\sqrt{1+e^x}}\,dx$

   **(m)** $\displaystyle\int x\,\sin^2 x\,\cos x\,\,dx$

   **(n)** $\displaystyle\int \sqrt{1-\sin x}\,dx$

# Math 202

# Assignment 13: Improper Integral

**The due date for this assignment is .....**
Reading assignment: Section 7.8


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1. Determine weather each integral is convergent or divergent. Evaluate those that are convergent.

   **(a)** $\displaystyle\int_0^{\infty} \sin^2\alpha \; d\alpha$

   **(b)** $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\, dx$

   **(c)** $\displaystyle\int_0^{3} \frac{dx}{x^2-6x+5}$

   **(d)** $\displaystyle\int_{-1}^{0} \frac{e^{1\backslash x}}{x^3}\, dx$

   **(e)** $\displaystyle\int_0^{2} x^2 \ln x \;\; dx$

2. Sketch the region and find its area (if the area is finite)

$$S = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq e^{-x}\}$$

3. Use the Comparison Theorem to determine weather the integral convergent or divergent.

   **(a)** $\displaystyle\int_0^{\infty} \frac{x}{x^3+1}\, dx$

   **(b)** $\displaystyle\int_1^{\infty} \frac{2+e^{-x}}{x}\, dx$

   **(c)** $\displaystyle\int_1^{\infty} \frac{x+1}{\sqrt{x^4-x}}\, dx$

**(d)** $\displaystyle\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$

**(e)** $\displaystyle\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx$

**(f)** $\displaystyle\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$

4. The integral

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}dx$$

is improper for two reasons: The interval $[0,\infty)$ is infinite and the integrand has an infinite discontinuity at 0. Evaluate it by expressing it as a sum of improper integrals of Type 2 and Type 1 as follows:

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}dx = \int_0^1 \frac{1}{\sqrt{x}\,(1+x)}dx + \int_1^\infty \frac{1}{\sqrt{x}\,(1+x)}dx$$

# Math 202

# Assignment 14: Areas Between Curves

**The due date for this assignment is .....**
Reading assignment: Section 6.1


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1. Sketch the region enclosed by the given curves. Decide weather to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

   **(a)** $y = \dfrac{1}{x}, \qquad y = \dfrac{1}{x^2}, \qquad x = 2$

2. Sketch the region enclosed by the given curves and find its area.

   **(a)** $y = e^x, \quad y = xe^x, \quad x = 0.$

   **(b)** $y = \tan x, \quad y = 2\sin x, \quad -\pi\backslash 3 \leq x \leq \pi\backslash 3$

   **(c)** $y = \cos x, \quad y = 1 - \cos x, \quad 0 \leq x \leq \pi$

   **(d)** $y = \dfrac{1}{x}, \quad y = x, \quad y = \dfrac{1}{4}x, \quad x > 0$

3. Use calculus to find the area of the triangle with the given vertices.

   **(a)** $(0,0)\,, \quad (3,1)\,, \quad (1,2)$

# Math 202

# Assignment 15: Volumes

**The due date for this assignment is .....**
Reading assignment: Section 6.2


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1. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

   **(a)** $y = \ln x, \quad y = 1, \quad y = 2, \quad x = 0; \quad$ about the $y - axis$

   **(b)** $y = \frac{1}{4}x^2, \quad y = 5 - x^2; \quad$ about the $x - axis$

   **(c)** $y = \frac{1}{4}x^2, \quad x = 2, \quad y = 0; \quad$ about the $y - axis$

   **(d)** $y = 1 + \sec x, \quad y = 3, \quad$ about $y = 1$

   **(e)** $y = \sin x, \quad y = \cos x, \quad 0 \leq x \leq \pi\backslash 4; \quad$ about $y = -1$

   **(f)** $x = y^2, \quad x = 1; \quad$ about $x = 1$

   **(g)** $y = x, \quad y = \sqrt{x}; \quad$ about $x = 2$

# Math 202

# Assignment 16: Arc Length

**The due date for this assignment is .....**
Reading assignment: Section 8.1


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1. Find the exact length of the curve

   **(a)** $x = \frac{1}{3}\sqrt{y}\,(y-3)\,, \quad 1 \le y \le 9$

   **(b)** $y = \ln(\cos x)\,, \qquad 0 \le x \le \pi\backslash 3$

   **(c)** $y = 3 + \frac{1}{2}\cosh 2x, \qquad 0 \le x \le 1$

   **(d)** $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \qquad 1 \le x \le 2$

   **(e)** $y = \ln(1-x^2)\,, \qquad 0 \le x \le \frac{1}{2}$

2. Find the arc length function for the curve $y = 2x^{3\backslash 2}$ with starting point $P_0\,(1,2)\,.$

# Math 202

# Assignment 17: Area of Surface of Revolution

**The due date for this assignment is .....**
Reading assignment: Section 8.2


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1. Find the exact area of the surface obtained by rotating the curve about the $x-axis$

   **(a)** $9x = y^2 + 18, \quad 2 \le x \le 6$

   **(b)** $y = \sin \pi x, \qquad 0 \le x \le 1$

   **(c)** $y = \dfrac{x^3}{6} + \dfrac{1}{2x}, \qquad \dfrac{1}{2} \le x \le 1$

2. The given curve is rotated about the $y-axis$. Find the area of the resulting surface.

   **(a)** $y = \sqrt[3]{x}, \qquad 1 \le y \le 2$

   **(b)** $y = \dfrac{1}{4}x^2 - \dfrac{1}{2}\ln x, \qquad 1 \le x \le 2$